THE RELIABILITY HORIZON

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The *"reliability horizon"* for semi-classical quantum gravity quantifies the extent to which we should trust semi-classical quantum gravity, and gives a handle on just where the *"Planck regime"* resides. The key obstruction to pushing semi-classical quantum gravity into the Planck regime is often the existence of large metric fluctuations, rather than a large back-reaction.

1 Introduction

Semi-classical quantum gravity is the approximation wherein we keep the gravitational field classical, while quantizing everything else. Where should we stop believing this approximation? Qualitatively, the answer to this question has been known since the pioneering work of Wheeler. [1,2] We should certainly stop believing semi-classical quantum gravity once we enter the Planck regime. The subtleties arise in recognizing the onset of Planck scale physics.

To quantify these issues I have recently introduced the concepts of the *"reliable region"*, the *"reliability boundary"*, and the *"reliability horizon"*. [3] These concepts are defined in a way that is similar to standard concepts of classical general relativity: the chronology-violating region; chronology boundary; and chronology horizon respectively. Physically, the reliability horizon is characterized by the onset of *either* large metric fluctuations (Planck scale curvature fluctuations) *or* a large back-reaction (Planck scale expectation value for the curvature). In many situations the onset of large metric fluctuations precedes the onset of large back-reaction. [3,4]

2 The chronology horizon

Let γ be any geodesic (spacelike, null, or timelike) that connects some point x to itself. Let $\sigma_\gamma(x,y)$ denote the relativistic interval from x to y along the geodesic γ.

$$\sigma_\gamma(x,y) = \begin{cases} +s^2 & \text{if the geodesic is spacelike,} \\ 0 & \text{if the geodesic is lightlike,} \\ -\tau^2 & \text{if the geodesic is timelike.} \end{cases} \tag{1}$$

Define level sets $\Omega(\ell^2)$ by

$$\Omega(\ell^2) \equiv \left\{ x : \exists\gamma \neq 0 | \sigma_\gamma(x,x) \leq \ell^2 \right\}. \tag{2}$$

—The set $\Omega(0)$ is called the *"chronology violating region"*.
—The set $\mathcal{B} \equiv \partial[\Omega(0)]$ is the *"chronology boundary"*. This is the boundary that we will have to cross in order to actively participate in time travel effects.
—The set $\mathcal{H}^+ \equiv \partial[J^+(\Omega(0))]$ is the *"chronology horizon"*. It is the boundary of the

future of the chronology violating region. This is the boundary that we will have to cross in order to passively participate in time travel effects.

These definitions only make sense if we are dealing with a fixed uquantized Lorentzian geometry—this is exactly the statement that we are dealing with semi-classical quantum gravity.

3 The reliability horizon

I define the reliability horizon in three stages: [3]

Definition 1a: Using notation as above, let $\mathcal{U} \equiv \Omega(+\ell^2_{Planck})$ be the *"unreliable region"*. It consists of those points x that are connected to themselves by spacelike geodesics as short as, or shorter than, one Planck length.

The entire thrust of this definition is that it gives an invariant and unambiguous meaning to the notion "within a Planck length of the chronology horizon", an invariant interpretation of this phrase being necessary before it is possible to decide where the Planck regime resides.

Definition 1b: Let $\mathcal{B}_{Planck} \equiv \partial[\mathcal{U}] \equiv \partial[\Omega(+\ell^2_{Planck})]$ be the *"reliability boundary"*—this is the boundary that we will have to cross in order to actively probe the unreliable region.

Definition 1c: The set $\mathcal{H}^+_{Planck} \equiv \partial[J^+(\mathcal{U})] \equiv \partial[J^+(\Omega(+\ell^2_{Planck}))]$ is the *"reliability horizon"*—it is the boundary of the future of the unreliable region. This is the boundary that we will have to cross in order to passively probe the unreliable region.

Justification: At this stage these are merely definitions, they do not carry any weight until we physically justify the terminology. [3] If we take Einstein gravity and linearize it about the background we are interested in, we can then ask how the linearized gravitons behave as quantum fields on this background geometry. The resulting quantum field theory is well known to be non-renormalizable with a dimensionful coupling constant given by the Planck mass. [1, 2] Once we enter the unreliable region these linearized gravitons are subject to Planck scale physics which in this non-renormalizable theory is definitely a disaster. Inside the unreliable region the linearized gravitons will be strongly interacting (and also unitarity violating) and will thereby lead to Planck scale fluctuations in the curvature, [3] even if the expectation value of the curvature is pleasingly mild. [4] It is these large metric fluctuations and associated Planck scale curvature fluctuations that tell us that we should no longer trust semi-classical quantum gravity behind the reliability boundary. [3]

Definition 2: An improvement of the previous definition, if the manifold in question is multiply connected one, is to keep track of the winding number of the geodesic. (In spacetimes containing traversable wormholes [5, 6] this will just be the number of times the geodesic threads through one of the wormholes.) Decompose the homotopy classes of self-intersecting geodesics emanating from the point x into equivalence classes Γ_N characterized by winding number N, and define

$$\Omega_N(\ell^2) \equiv \left\{ x : \exists \gamma \in \Gamma_N | \sigma_\gamma(x,x) \leq N^2\ell^2 \right\}. \tag{3}$$

The point is that if a geodesic wraps through N wormholes and is of length less than $N\ell$, then at least one wormhole-to-wormhole segment of the curve must be of

length less than ℓ. Now simply replace $\Omega(\ell^2)$ by

$$\Omega_\infty(\ell^2) \equiv \Omega(\ell^2) \cup (\cup_{N=1}^{\infty} \Omega_N(\ell^2)) \tag{4}$$

in all definitions regarding the reliability region.

Definition 3: The definition given above still does not capture all of the situations in which we should cease trusting semi-classical quantum gravity. We should also not trust regions where the background manifold exhibits Planck scale curvature. We might naively decide to look at sets such as

$$\Omega_R(\ell^2) = \left\{ x : R_{\mu\nu\sigma\rho} R^{\mu\nu\sigma\rho} > \ell^{-2} \right\}. \tag{5}$$

We now augment the unreliable region by setting

$$\mathcal{U} \equiv \Omega_\infty(+\ell^2_{Planck}) \cup \Omega_R(+\ell^2_{Planck}). \tag{6}$$

Similarly, the reliability boundary becomes

$$\mathcal{B}_{Planck} \;\; \equiv \;\; \partial[\mathcal{U}] \;\; \equiv \;\; \partial\left[\Omega_\infty(+\ell^2_{Planck}) \cup \Omega_R(+\ell^2_{Planck})\right], \tag{7}$$

and the reliability horizon becomes

$$\mathcal{H}^+_{Planck} \;\; \equiv \;\; \partial[J^+(\mathcal{U})] \;\; \equiv \;\; \partial[J^+\left(\Omega_\infty(+\ell^2_{Planck}) \cup \Omega_R(+\ell^2_{Planck})\right)]. \tag{8}$$

4 Discussion

The reliability boundary has important applications to Hawking's chronology protection conjecture, [7,8,9] and to the related Kay–Radzikowski–Wald singularity theorems. [10,11] I argue [3] that the chronology horizon is always hidden by the reliability horizon and that semiclassical quantum gravity should fail before reaching the chronology horizon.

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